STEP-POOL MORPHOLOGY IN HIGH-GRADIENT STREAMS

A.R. MAXWELL[1] and A.N. PAPANICOLAOU[2]

ABSTRACT

The focus of this study is to examine bed stability and morphology in high-gradient gravel-bed streams, and thus to improve understanding of the various parameters governing the sediment flow characteristics in mountain streams. Ultimately, this knowledge can be used to design pseudo-natural channels, as in the stream simulation method of culvert design; with this in mind, prototype conditions are evaluated in a flume with slopes ranging from 3% to 7%, and particle relative submergence varying from 0.5 to 2.5 for three bed size distributions. These experiments are designed to satisfy the conditions of dynamic similarity for flow and sediment, and they are preferred over field measurements since they allow a high degree of control over testing conditions. It is found that step-pool bedforms are the most ubiquitous features along the gravel bed. A new formula is developed that correlates step height with the gravel-bed size distribution, relative submergence of the particles, and the Froude number. The step spacing is found to be related to step height and streambed longitudinal slope. Flow resistance is also examined, and a formula is developed which accounts for the resistance due to the bedforms (form resistance), as well as the individual sediment particles (grain resistance).

Key Words: Mountain streams, Step-pool bedforms, Step height

1 INTRODUCTION

Gravel bed rivers are found in many parts of the world, typically in mountainous regions with high gradients and seasonally high flows. These rivers are important in controlling flood waters from spring runoff in regions such as the Pacific Northwest, where heavy snowfall can be followed by equally heavy rainfall. The combination of high stream gradient and high discharge causes significant erosion of the bed and bank of the stream, in some cases moving large boulders with ease.

It is desirable to predict the hydraulic and geomorphologic interactions, which can ultimately be used to minimize or prevent detrimental effects on stream ecology and infrastructure such as bridges and culverts. Various researchers have noted the presence of bedforms in high-gradient, gravel-bed rivers (e.g. Billi et al., 1998; Whittaker, 1987). The overall morphology of a channel can be classified as pool, riffle, coarse (boulder) riffle, step-pool, or waterfall (Padmore et al., 1998). Such classifications are correlated with the local stream gradient, with pool-riffle sequences observed at slopes $S < 0.02$, coarse riffles or cascades at slopes ranging from 0.03 – 0.07, and step-pools as the dominant feature at slopes of 0.04 - 0.20 (Grant et al., 1990). The observed ranges for step-pool by Grant et al. (1990) are consistent with the reports of Whittaker and Jaeggi (1982) and Billi et al. (1998), who, investigated step-pool formation in a laboratory flume and a field study, respectively. The geometric characteristics of step-pool formations and their affects on flow are the focus of the present study.

1.1 Geometric Features of Step-Pools

Step-pool morphology is defined by a regular series of steps, similar to a staircase in the bed of the stream (Fig. 1a, b). Steps can be formed of large woody debris (Duckson and Duckson, 1995), and some researchers have observed step-pool geometry in bedrock channels (Grodek et al., 1994), formed similarly to those found in alluvial channels. The most common step type, however, is composed of the coarsest fraction of bed material interlaced with fines (Billi et al., 1998). The pool regions, which typically have an adverse slope, (Abrahams et al., 1995) also provide storage for finer bed material.

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Grant et al. (1990) conducted field studies of step-pool channel morphology in the Cascade mountains of Oregon. In an effort to separate the various channel configurations, characterizations were made based on the length of feature separation, in units of stream width. This allows a distinction between a "channel unit sequence" of stepped-bed morphology (1 – 2 times the channel width between steps), defined as a series of small steps between larger pools, and the "step-pool sequences" (0.1 – 1 times the channel width between steps) which are separated by local pools. The confusion of terminology in the extant literature is evident as various authors discuss pool-pool spacing, step spacing, pool depth, pool length, and step height, with different meanings often attributed to the same term.

Fig. 1a Typical step-pool configuration

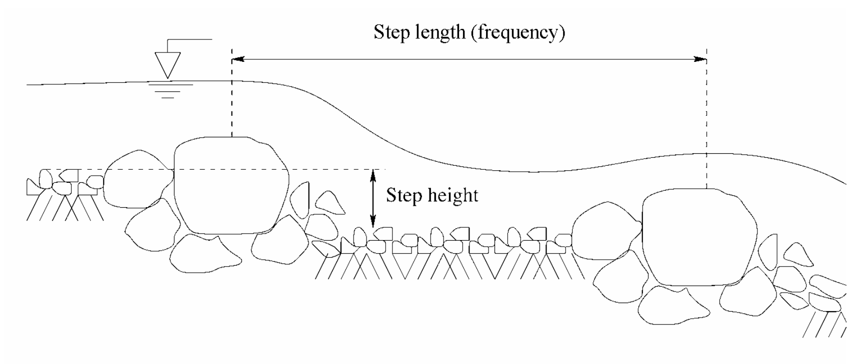

Fig. 1b Schematic of step-pool configuration

1.2 Prediction of Step-Pools

A number of theories have been propounded for the formation of step-pools, as researchers have attempted to explain the existence of bedforms in gravel streams. One of the earliest propositions was that step-pool systems form in response to a standing wave, behaving as an antidune bedform (Whittaker and Jaeggi, 1982). Whittaker and Jaeggi's work showed that their flume data were consistent with the antidune model for slopes less than 0.075, and they hypothesized that the same mode of formation exists at higher slopes, but is modified by the flow and large roughness elements.

Abrahams et al. (1995), as well as most other researchers (e.g., Whittaker and Jaeggi, 1982; Billi et al., 1998), took step length L (or, equivalently, spacing between two subsequent steps) and step height d_{step} (the step height in Abrahams et al. is denoted as H) as the best overall characterization of step-pool form. The Abrahams et al. study examined the relation between L, d_{step}, and channel slope, S, but did not develop a relationship to predict actual height and length of steps and pools. In order to make observations of this type, steps must be allowed to form naturally, i.e., by a formative discharge.

Further complicating the hydraulic characteristics of a step-pool bedform is the lack of reliable formulas to determine the stage-discharge relationship. Although many formulas exist to predict the friction factor of mild-sloped streams (e.g., the well-known Strickler relationships), the frictional characteristics of mountain streams are not always well-represented by these relationships (Thorne and Zevenbergen, 1985).

In the present study, a physical model was deemed the most effective means to investigate the interaction of the flow and sediment, as this allows detailed measurements to be made in a well-controlled environment. On the basis of the model testing, this research enhances the state-of-the-art knowledge in the area of river morphology in mountain streams; based on that knowledge, the present study provides relationships to predict (i) step height, (ii) step spacing, (iii) bed stability, and (iv) frictional characteristics of step-pool bedforms. Practical applications include culvert retrofit, hydraulic design (e.g. flood control), and stream restoration. The proposed relationships are developed for slopes ranging from 3% - 7% and relative submergence ranging from 0.5 – 2.5.

2 METHODOLOGY

The backbone of the proposed methodology is founded on the latest findings in the area of sediment transport for mountain streams. Various authors, e.g., Bettess (1999); Suszka (1991); and Bathurst (1985), have clearly demonstrated that sediment motion and flow frictional characteristics in mountain streams are dependent on the relative submergence of the particle, defined as the ratio H/D_{84}, where H is flow depth and D_{84} is the 84th percentile median axis particle diameter (i.e., 84 percent of the material is finer than D_{84}). In this case, the incipient motion stress (the bed shear stress necessary to dislodge a particle) is no longer dependent on the particle Reynolds number but on the relative submergence and the Froude number (Peakall et al., 1996; Kilgore and Young, 1993).

The design procedure developed here for streambeds is based on the consideration that step-pool configurations are ubiquitous features of mountain streams. Furthermore, when step-pool bedforms are present at an equilibrium stage, they will have a stabilizing effect on the bed since they dissipate the kinetic energy of fast moving water (Wohl, 2000). Knowledge of the characteristics of such bedforms is necessary to predict stable step-pool bedforms, for example in a restoration project.

A significant component of this research was devoted to the scaling of the experiments. In order to ensure the validity of the model study proposed here, it was necessary to satisfy the scaling laws of dynamic similitude for flow and sediment. In the present study, use of the Froude number was necessary in order to satisfy the conditions of dynamic similarity for flow and sediment. Since flow on steep slopes is driven by gravity, the incipient motion stress is strongly dependent on the Froude number (Kilgore and Young, 1993). The full derivation of the scaling methodology is not the focus of this article and has been presented in detail in a recent publication by Maxwell and Papanicolaou (1999).

3 EXPERIMENTAL SETUP

3.1 Facility

The testing for the present study was performed in the R.L. Albrook Hydraulics Laboratory of Washington State University. The primary test apparatus was a tilting, water-recirculating flume (Fig. 2) which is 21 m (70 ft) long, 0.91 m (3 ft) wide, and 0.61 m (2 ft) deep. One side of the flume was transparent acrylic, which permits side viewing of the flow, and the slope of the flume was variable from 0-14%. Water flow was provided by pumping from a large, semi-enclosed sump, located under the flume, and volumetric flowrate was measured with a magnetic flowmeter. A total station instrument was used to measure bed height changes, and straightedges were used to measure flow depths.

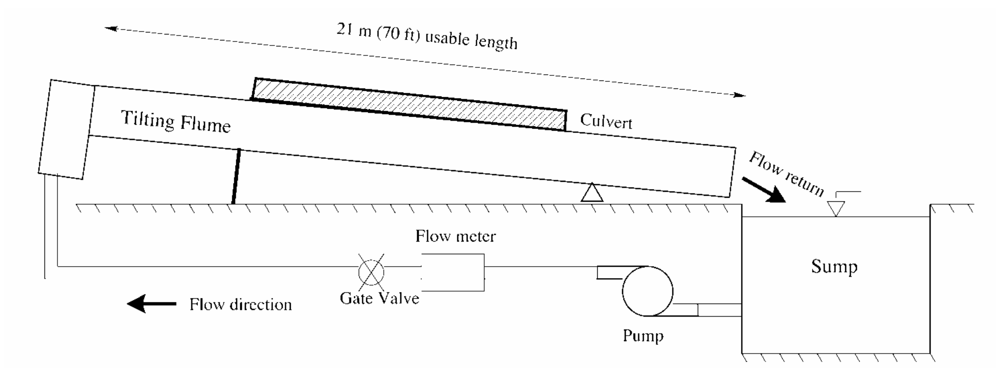

Fig. 2 Flume schematic (not to scale)

3.2 Sediment

To more effectively represent a generalized field condition, a mixture of sediment sizes was employed in testing. Natural river gravel of 6.4mm, 19mm, 38mm, and 64mm nominal sizes was obtained from local sources in approximately equal parts of each size class. The actual sizes ranged from less than 6mm to 76mm diameter, resulting in the grain size distributions shown in Fig. 3. Cobble size river rock was also added to the mixture as necessary to produce the grain size distributions shown in Fig. 3. All sizes were thoroughly mixed before and after placement in the flume, to ensure uniformity of the bed composition.

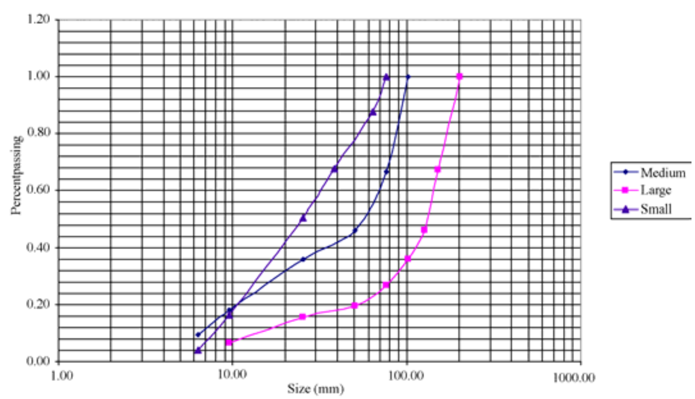

Fig. 3 Sediment size distributions tested

3.3 Procedure

Each test series with a specific sediment size distribution and bed slope was commenced by thoroughly mixing the sediment in the bed to minimize stratification; following this, the centerline of the bed was surveyed with the total station. Sediment gradation was measured using a standard mass sieving procedure. Water flow was then increased gradually, to prevent the bed from being eroded immediately by an initial wave of water, and flow was then fixed at a given relative submergence H/D_{84}. Flow depth for the tests was determined by averaging a series of measurements taken along the bed with a straightedge; in this manner, readings were taken in pools and on steps. The test was terminated when stable step-pool bedforms (*viz.*, bedforms whose height and spacing along the longitudinal direction does not change significantly during their migration (Wohl, 2000) were developed, or if massive erosion (failure) was evident. Some degree of subjectivity is inherent in these determinations, but the initial tests were repeated until a consistent methodology was established and repeatability was observed (Maxwell, 2000). Following termination of a test, the bed was again surveyed and the next flow scenario was run. When failure was observed, the next slope/sediment combination was tested, repeating the established procedure.

4 EXPERIMENTAL RESULTS

Step-pool formation was present at all slopes tested (3%, 5%, and 7%), and the bedforms were profound, especially, at low flows (Fig. 4a, 4b). In Fig. 4a, steps are visible in the low flow condition, looking upstream. The step-pool configuration forms naturally, over a period, which varies depending on flow, slope conditions, and bed sediment distribution. It is a "stable" bedform, i.e., the larger clasts tend to cluster together and shelter the smaller, more mobile particles; this is evident in Fig. 4b, which is the upstream portion of a step formation. In this case the spacing and step height of these bedforms tend to approach a pseudo–equilibrium condition, defined previously. Quantitative information on step height, bed stability, step spacing, and frictional characteristics of step-pool bedforms are provided next.

Fig. 4a Typical step-pool formation at 5% slope

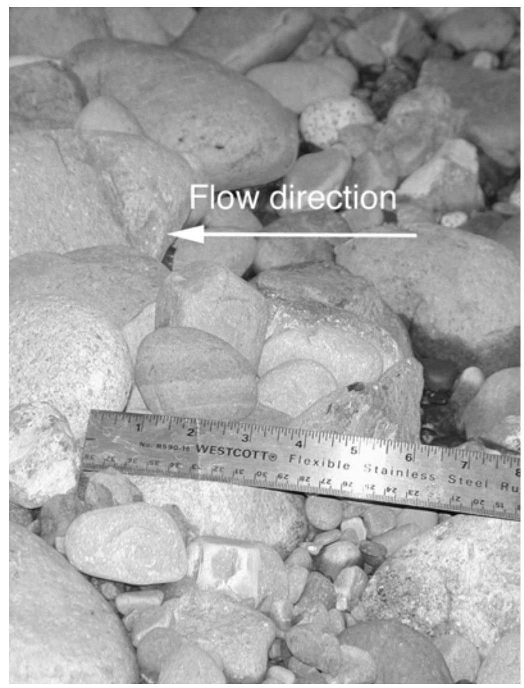

Fig. 4b Step structure, flow is from right to left

4.1 Step Height

Formation of step-pools in a streambed would have a stabilizing effect on the bed, as observed during testing of the model. Generally, the frequency of steps increases directly with flowrate, and decreases inversely with slope. The height of steps increases directly with flowrate, until a plane bed known as upper regime (Chang, 1988) is reached, with no bedforms; this is considered a failure condition. Some movement of steps was observed, but it was generally difficult to conclude a direction of migration; however, Whittaker and Jaeggi (1982) noted that step-pool formation appeared consistent with antidune behavior, which implies upstream migration of the standing wave. Such steps will form naturally, although sufficient upstream sediment supply is necessary; for the model, this was supplied by the gravel bed upstream of the measuring region, and tests were terminated before artificial scour occurred. Prediction of the equilibrium step height may be accomplished using the following dimensionless formula, Eq. (1), developed by Maxwell et al. (in press),

$$\frac{d_{step}}{H}\sigma^{0.5} = 2.0\left[\frac{Q}{\sqrt{gH^5}}\left(\frac{D_{50}}{H}\right)^{1.5}\right]^{0.31} \quad (1)$$

where d_{step} is step height, $\sigma = \sqrt{D_{84}/D_{16}}$ denotes the geometric standard deviation of the sediment size distribution, Q is volumetric flowrate, g is gravitational acceleration, and D_{50} is the 50th percentile median axis particle diameter. The relationship is shown graphically in Fig. 5, plotted with the data acquired from the model, and was developed for slopes ranging from 3% - 7% and relative submergence ranging from 0.5 – 2.5.

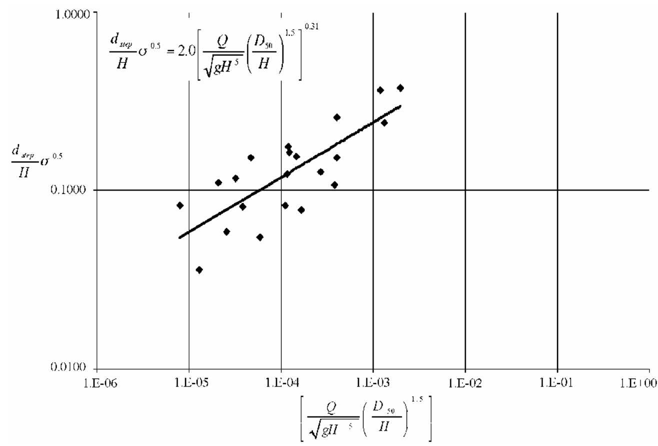

Fig. 5 Step height

4.2 Stability

To ensure sufficient particle size for bed stability, Eq. (1) must be used in conjunction with a stability relationship. If Eq. (1) is extrapolated outside of the range of values examined in testing, it may predict a bedform when, in fact, failure will occur. The relation suggested by Maxwell (2000) accounts for the dependence of the critical shear stress on Froude number, present in high-gradient streams, and is

$$\tau^*_{cr} = 0.03\boldsymbol{F}^{1.29} \quad (2)$$

where $\boldsymbol{F}$ is the Froude number and τ^*_{cr} is the dimensionless shear stress parameter given in Eq. (3), commonly referred to as the Shields parameter or critical stress.

$$\tau^*_{cr} = \frac{\gamma HS}{(\gamma_s - \gamma)D_{84}} \quad (3)$$

In the preceding equation, S is stream longitudinal slope, γ_s is sediment specific weight, γ is specific weight of water, and other variables have been previously defined. Typically, the values of the Froude number in this study were between 0.8 and 1.3, using the average velocity and depth over the length of the test region. A formula similar to Eq. (2) was developed by Kilgore and Young (1993) for riprap stability, but Eq. (2) provides a more conservative critical value than that of Kilgore and Young (1993) since it is developed for streambed stability. The same limitations of slope and relative submergence apply as to Eq. (1).

4.3 Step Spacing

Prediction of step spacing remains an open case; it has been shown by some researchers (e.g. Whittaker, 1987; Grant et al., 1990) to be dependent on bed slope, while a few researchers (e.g. Billi et al., 1998) have recently suggested that step spacing is a function of slope and step height. Maxwell et al. (2001) present the following Eq. (4), which is a composite of laboratory data from the present study and field data from Billi et al. (1998)

$$L = 7.39 \ln\left(\frac{d_{step}}{S}\right) - 5.52 \tag{4}$$

where L is step spacing in meters, and d_{step} is step height in meters, determined via Eq. (1). Eq. (4) suggests that step spacing is implicitly dependent on slope, sediment size distribution, and discharge (via Eq. (1)). The curve fit is shown in Fig. 6. This formula is applicable to streams of width varying within the range 4.8 – 7 m. Expansion of Eq. (4) to wider flow ranges and various rivers will be considered in the future. For the present, a survey conducted by the first author on the Big Quilcene River in Washington State (Fig. 1a) yielded extremely close agreement with (4). Measured slope was 4.4%, step height was 0.30 m, and the measured step spacing was 8.8 m; the calculated value of L = 8.7 m is very close to the measured value.

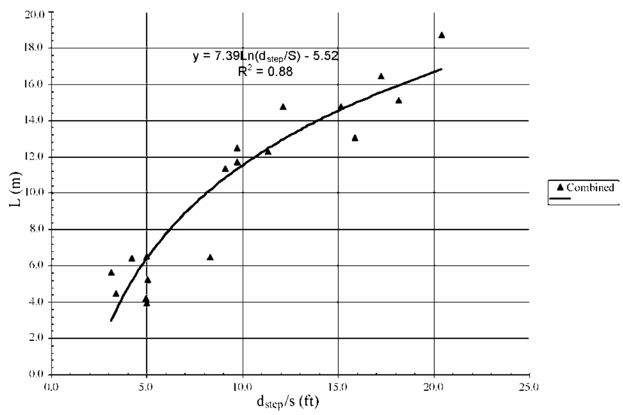

Fig. 6 Step spacing

4.4 Frictional Characteristics

In predicting the flow of water over a rough surface, it is necessary to quantify the roughness of the surface and relate its magnitude to the hydraulics of the flow. In the present study and others, it is evident that the high gradients typically result in near-bed flows that are chaotic in nature (transitional, not fully developed flows) (Rice et al., 1998). It is further evident that the step-pool provides form as well as grain resistance to flow (e.g. Whittaker and Jaeggi, 1982), but this has not been previously considered functionally for step-pool bedforms. Engelund (1966) investigated the hydraulic resistance of alluvial bedforms, and presented energy slope as a functional relationship of dune dimensions (amplitude and spacing). The work of Scheuerlein (1973) indicated that flow resistance was strongly dependent on spacing and amplitude of roughness elements, based on experimentation in channels of slopes up to 67%; later work by Griffiths (1981), Shen et al. (1990), and Millar (1999) bears out this functional relationship. However, no unifying formula has been found that allows prediction of frictional resistance in a generic sense, and the authors are unaware of any formula which claims to predict the friction factor of step-pool streams.

The Darcy-Weisbach formula, typically used to predict behavior of closed-conduit flows, can also be used for an open channel, in the following form:

$$f = \frac{8gRS}{U^2} \tag{5}$$

where f is the friction factor, g is gravitational acceleration, and other variables are as previously defined. This frictional relationship is nondimensional, and applicable to any open channel flow situation. However, the dependence of f must be established and empirically correlated to measurable conditions in a high-gradient stream.

The relative submergence of the bed particles has been shown experimentally to be a significant parameter when calculating f in high-gradient streams (Bathurst, 1985); it is noted that this is essentially the parameter with which one calculates f for closed conduit flow based on the Moody diagram (e.g.

Crowe et al., 2001). The equation developed by Bathurst (1985) based on the standard semilogarithmic resistance relationship, is

$$\sqrt{\frac{8}{f}} = 5.62\log\left(\frac{R}{D_{84}}\right) + 4 \tag{6}$$

where R is hydraulic radius and D_{84} is the 84th percentile median axis stone diameter. This formula is based on field data from streams ranging in slope from 0.004 – 0.0364, with bed material sizes ranging from 113 ≤ D_{84} ≤ 740 mm; consequently, it would not be applicable to streams with step-pool morphology, which generally occur at steeper slopes (Bathurst, personal communication, 2001). Thorne and Zevenbergen (1985) reviewed several flow resistance relationships, including Bathurst's, and noted errors of up to 30% in all of the equations tested.; consequently, they recommended that caution be used when applying any such equation, and that the predictive capabilities be checked with field data for a given site, if at all possible.

The limitation of such formulas as (6) is evident when one examines the independent variables found in those formulas; from a purely functional standpoint, (6) only accounts for grain resistance of the bed particles. Work previously cited indicates that this is not adequate in the presence of bedforms. Rather, it is proposed that flow resistance in step-pool streams is dependent on the following variables:

$$f = \phi\left(D_{84}, d_{step}, L\right) \tag{7}$$

where d_{step} is the height of a step-pool bedform, and L is spacing between step peaks (Millar, 1999). Millar (1999) used the well-established concept of separating the energy slope on the bed into form and grain components as in (8)

$$S = S' + S'' \tag{8}$$

where S' is energy loss due to grain resistance (i.e., individual particles as roughness elements), and S'' is energy loss due to form resistance (by the pressure difference between the front and rear end of a bedform). Engelund (1966), as well as Griffiths (1981) and Shen et al. (1990) also employed this concept in their examination of flow resistance. To date, however, no attempts have been made to correlate form resistance in step-pool streams with a frictional factor; this paper employs the basic methodology and concepts presented to develop a resistance formula for step-pool streams. The parameters in (7) may be arranged in a dimensionless form as:

$$\sqrt{\frac{8}{f}} = \phi\left(\frac{d_{step}D_{84}}{LH}\right) \tag{9}$$

where the bedform slope is multiplied by the relative roughness due to the sediment. The resistance factor is plotted against the log of the functional relationship presented in (9), with results, which show an evident trend (Fig. 7). A linear regression fit performed using the computer software IGOR Pro resulted in the following formula:

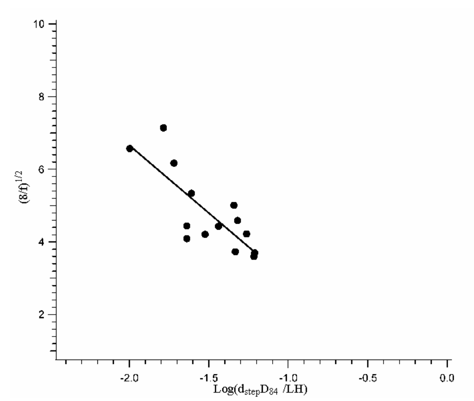

Fig. 7 Friction factor regression analysis

$$\sqrt{\frac{8}{f}} = -3.73\log\left(\frac{d_{step}D_{84}}{LH}\right) - 0.80 \tag{10}$$

This formula is dimensionless, and enables the prediction of the Darcy friction factor, based on form and grain roughness of a gravel bed. Following the methodology of Millar (1999), the skin friction f' was then calculated using the Keulegan relation by setting the bed roughness equal to the median diameter

$$f' = \left(2.03\log\left(\frac{12.2H}{D_{50}}\right)\right)^{-2} \tag{11}$$

and the grain friction slope S' was then calculated using the Darcy Eq. (5); rearranging (8), then, the form friction slope S'' was computed. Fig. 8 shows the relation of S'' to step steepness d_{step}/L; the best-fit line indicates that $S'' = 0.75(d_{step}/L)$, which agrees with the suggested notion that form roughness is the dominant type of roughness in high-gradient streams with profound bedform features. Millar (1999) suggested that $S'' = 0.95(d_{step}/L)$. The different value of S'' obtained by Millar's (1999) analysis is attributed to the fact that the latter study focused on frictional characteristics of pool-riffle stream data under bankfull or nearly full discharge; in this case, grain roughness is "unseen" by the flow, while in the step-pool cases under consideration in the present study, the lower relative submergence places higher weight on the grain resistance. Fig. 9 compares the total friction factor predicted by the present study Eq. (10), along with the total friction factor predicted by the Keulegan method; the predictive capabilities appear to be nearly equal, but the methodology of (10) is simpler for stage-discharge calculations. The limits of applicability for Eq. (9) are the same as for Eq. (1). It should also be noted that the well-known Manning equation can be used in conjunction with Eq. (5) to replace the Darcy friction factor with an equivalent Manning's *n*-value.

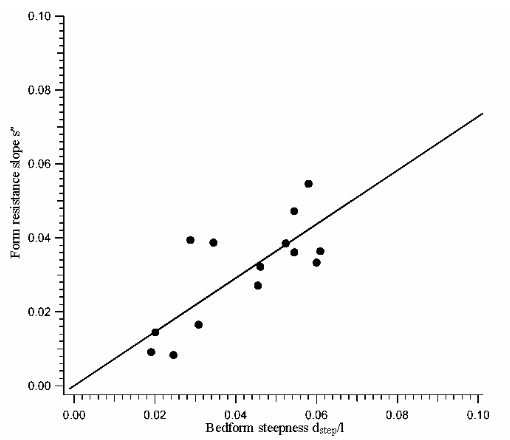

Fig. 8 Form resistance slope vs. bedform steepness

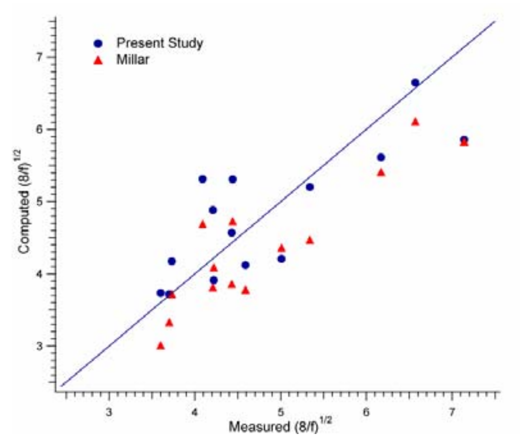

Fig. 9 Comparison of present formula and Millar formula

6 CONCLUSIONS

This work is the first to present a unified method of determining geometric and hydraulic characteristics of step-pool streams, including flow resistance based on bedform drag. In the present research, the generic findings of several flume experimental data related to the geometric and hydraulic characteristics of step-pool streams were further analyzed to develop formulas that predict the spatial and temporal characteristics of step-pool bedforms.

As such, formulas, which enable:

- Prediction of step height
- Prediction of step spacing
- Prediction of flow resistance

for streams with profound step-pool bedforms were developed by satisfying the dynamic similitude conditions for flow and sediment. The predictions of these formulas compare well with other field and laboratory data for the flow and sediment conditions tested here. While the current study advances the knowledge of step-pool formation and stability and provides a unified approach for predicting step height and step spacing when equilibrium conditions are present, further testing or calibration of the suggested formulas is necessary to enable their application in practical stream restoration design projects.

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